

# Database Systems

## Lecture #4

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# **Chapter 3: Relational Algebra and Calculus.....to be continue**

# Queries

- **A query** is a function from database instances to relations.
- Queries are formulated in relational algebra by means of ***expressions over relations***.

# A database for the examples

## EMPLOYEES

Number	Name	Age	Salary
101	Mona Salem	34	40
103	Mona Ahmed	23	35
104	Amr Khalil	38	61
105	Ihab Zohdy	44	38
210	Medhat Aly	49	60
231	Samia Elkholy	50	60
252	Ihab Zohdy	44	70
301	Samir Salem	34	70
375	Mona Salem	50	65

## SUPERVISION

Head	Employee
210	101
210	103
210	104
231	105
301	210
301	231
375	252

# Example1:

- **Write a relational algebra expression for the following query:**
  - Find the numbers, names and ages of employees earning more than 40 thousands.

$$\pi_{\text{Number, Name, Age}} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES}))$$

Number	Name	Age
104	Amr Khalil	38
210	Medhat Aly	49
231	Samia Elkholy	50
252	Ihab Zohdy	44
301	Samir Salem	34
375	Mona Salem	50

# Example 2

- **Write a relational algebra expression for the following query:**
  - Find the registration numbers of the supervisors of the employees earning more than 40 thousands.

$\pi_{\text{Head}} (\text{SUPERVISION} \bowtie_{\text{Employee} = \text{Number}} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES})))$

Head
210
301
375

## Answer Ex.2 in details:

### Step1:

$\sigma_{\text{Salary} > 40}(\text{EMPLOYEES}) = \mathbf{R1}$

Number	Name	Age	Salary
104	Amr Khalil	38	61
210	Medhat Aly	49	60
231	Samia Elkholy	50	60
252	Ihab Zohdy	44	70
301	Samir Salem	34	70
375	Mona Salem	50	65

### Step2:

**SUPERVISION**  $\bowtie$  **Employee = Number (R1)**

Head	Employee	Number	Name	Age	Salary
210	104	104	Amr Khalil	38	61
301	210	210	Medhat Aly	49	60
301	231	231	Samia Elkholy	50	60
375	252	252	Ihab Zohdy	44	70

### Step3:

$\pi_{\text{Head}}(\text{SUPERVISION} \bowtie \text{Employee = Number (R1)})$

Head
210
301
375

**The  
Result**

# Example 3

- **Write a relational algebra expression for the following query:**
  - Find the names and salaries of the supervisors of the employees earning more than 40 thousands.

$$\pi_{\text{NameH}, \text{SalaryH}} (\rho_{\text{NumberH}, \text{NameH}, \text{AgeH}, \text{SalaryH} \leftarrow \text{Number}, \text{Name}, \text{Age}, \text{Salary}} (\text{EMPLOYEES}) \\ \bowtie_{\text{NumberH}=\text{Head}} \\ (\text{SUPERVISION} \bowtie_{\text{Employee} = \text{Number}} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES}))))$$

NameH	SalaryH
Medhat Aly	60
Samir Salem	70
Mona Salem	65



# Answer Ex.3 in details:

$\rho_{\text{NumberH, NameH, AgeH, SalaryH} \leftarrow \text{Number, Name, Age, Salary}} (\text{EMPLOYEES}) = \text{R1}$

$\sigma_{\text{Salary} > 40} (\text{EMPLOYEES})$

Number	Name	Age	Salary
104	Amr Khalil	38	61
210	Medhat Aly	49	60
231	Samia Elkholy	50	60
252	Ihab Zohdy	44	70
301	Samir Salem	34	70
375	Mona Salem	50	65

NumberH	NameH	AgeH	SalaryH
101	Mona Salem	34	40
103	Mona Ahmed	23	35
104	Amr Khalil	38	61
105	Ihab Zohdy	44	38
210	Medhat Aly	49	60
231	Samia Elkholy	50	60
252	Ihab Zohdy	44	70
301	Samir Salem	34	70
375	Mona Salem	50	65

$(\text{SUPERVISION} \bowtie \text{Employee} = \text{Number} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES}))) = \text{R2}$

Head	Employee	Number	Name	Age	Salary
210	104	104	Amr Khalil	38	61
301	210	210	Medhat Aly	49	60
301	231	231	Samia Elkholy	50	60
375	252	252	Ihab Zohdy	44	70

# Answer Ex.3 in details (Cont.):

**R1**  **NumberH=Head** **R2**

NumberH	NameH	AgeH	SalaryH	Head	Employee	Number	Name	Age	Salary
<b>210</b>	Medhat Aly	49	60	<b>210</b>	104	104	Amr Khalil	38	61
<b>301</b>	Samir Salem	34	70	<b>301</b>	210	210	Medhat Aly	49	60
<b>301</b>	Samir Salem	34	70	<b>301</b>	231	231	Samia Elkholy	50	60
<b>375</b>	Mona Salem	50	65	<b>375</b>	252	252	Ihab Zohdy	44	70

$\pi_{\text{NameH, SalaryH}} (\mathbf{R1} \bowtie \mathbf{NumberH=Head} \mathbf{R2})$

NameH	SalaryH
Medhat Aly	60
Samir Salem	70
Mona Salem	65



$\pi_{\text{NameH, SalaryH}} (\rho_{\text{NumberH, NameH, AgeH, SalaryH}} \leftarrow \text{Number, Name, Age, Salary} (\mathbf{EMPLOYEES}))$



**NumberH=Head**

$(\mathbf{SUPERVISION} \bowtie_{\text{Employee = Number}} (\sigma_{\text{Salary} > 40} (\mathbf{EMPLOYEES})))$

# Example 4

- Write a relational algebra expression for the following query:
  - Find the employees earning more than their respective supervisors, showing registration numbers, names and salaries of the employees and supervisors.

$$\pi_{\text{Number, Name, Salary, NumberH, NameH, SalaryH}} \left( \sigma_{\text{Salary} > \text{SalaryH}} \left( \rho_{\text{NumberH, NameH, SalaryH, AgeH} \leftarrow \text{Number, Name, Salary, Age}} (\text{EMPLOYEES}) \right. \right. \\ \left. \left. \bowtie_{\text{NumberH=Head}} (\text{SUPERVISION} \bowtie_{\text{Employee = Number}} (\text{EMPLOYEES})) \right) \right)$$

Number	Name	Salary	NumberH	NameH	SalaryH
104	Ahmed Khalil	61	210	Medhat Aly	60
252	Ihab Zohdy	70	375	Mona Salim	65

## Answer Ex.4 in details:

$\rho_{\text{NumberH, NameH, AgeH, SalaryH}} \leftarrow \text{Number, Name, Age, Salary} (\text{EMPLOYEES}) = \text{R1}$

NumberH	NameH	AgeH	SalaryH
101	Mona Salem	34	40
103	Mona Ahmed	23	35
104	Amr Khalil	38	61
105	Ihab Zohdy	44	38
210	Medhat Aly	49	60
231	Samia Elkholy	50	60
252	Ihab Zohdy	44	70
301	Samir Salem	34	70
375	Mona Salem	50	65

## Answer Ex.4 in details (Cont.): :

**SUPERVISION**  $\bowtie$  **Employee = Number (EMPLOYEES) = R2**

Head	Employee	Number	Name	Age	Salary
210	101	101	Mona Salem	34	40
210	103	103	Mona Ahmed	23	35
210	104	104	Amr Khalil	38	61
231	105	105	Ihab Zohdy	44	38
301	210	210	Medhat Aly	49	60
301	231	231	Samia Elkholy	50	60
375	252	252	Ihab Zohdy	44	70

## Answer Ex.4 in details (Cont.):

$R1 \bowtie_{\text{NumberH=Head}} R2$

NumberH	NameH	AgeH	SalaryH	Head	Employee	Number	Name	Age	Salary
<b>210</b>	Medhat Aly	49	60	<b>210</b>	101	101	Mona Salem	34	40
<b>210</b>	Medhat Aly	49	60	<b>210</b>	103	103	Mona Ahmed	23	35
<b>210</b>	Medhat Aly	49	60	<b>210</b>	104	104	Amr Khalil	38	61
<b>231</b>	Samia Elkholy	50	60	<b>231</b>	105	105	Ihab Zohdy	44	38
<b>301</b>	Samir Salem	34	70	<b>301</b>	210	210	Medhat Aly	49	60
<b>301</b>	Samir Salem	34	70	<b>301</b>	231	231	Samia Elkholy	50	60
<b>375</b>	Mona Salem	50	65	<b>375</b>	252	252	Ihab Zohdy	44	70

$\sigma_{\text{Salary} > \text{SalaryH}} (R1 \bowtie_{\text{NumberH=Head}} R2) = R3$

## Answer Ex.4 in details (Cont.):

NumberH	NameH	AgeH	SalaryH	Head	Employee	Number	Name	Age	Salary
<b>210</b>	Medhat Aly	49	60	<b>210</b>	104	104	Amr Khalil	38	61
<b>375</b>	Mona Salem	50	65	<b>375</b>	252	252	Ihab Zohdy	44	70

$\pi_{\text{Number, Name, Salary, NumberH, NameH, SalaryH}} (\mathbf{R3})$

**The  
Result**

Number	Name	Salary	NumberH	NameH	SalaryH
104	Amr Khalil	61	210	Medhat Aly	60
252	Ihab Zohdy	70	375	Mona Salem	65

# Example 5

- Write a relational algebra expression for the following query:
  - Find the registration numbers and names of the supervisors whose employees **all** earn more than 40 thousands.

$$\pi_{\text{Number, Name}} (\text{EMPLOYEES} \bowtie \text{Number=Head})$$

$$(\pi_{\text{Head}} (\text{SUPERVISION}) -$$

$$\pi_{\text{Head}} (\text{SUPERVISION} \bowtie \text{Employee = Number} (\sigma_{\text{Salary} \leq 40} (\text{EMPLOYEES}))))))$$

Number	Name
301	Samir Salem
375	Mona Salem



## Answer Ex.5 in details:

$\sigma_{\text{Salary} \leq 40}(\text{EMPLOYEES}) = R1$

Number	Name	Age	Salary
101	Mona Salem	34	40
103	Mona Ahmed	23	35
105	Ihab Zohdy	44	38

$\text{SUPERVISION} \bowtie_{\text{Employee} = \text{Number}} (R1)$

Head	Employee	NumberH	NameH	AgeH	SalaryH
210	101	101	Mona Salem	34	40
210	103	103	Mona Ahmed	23	35
231	105	105	Ihab Zohdy	44	38

$\pi_{\text{Head}}(\text{SUPERVISION} \bowtie_{\text{Employee} = \text{Number}} (R1))$

Head
210
231

## Answer Ex.5 in details (Cont.):

$$(\pi_{\text{Head}}(\text{SUPERVISION}) - \pi_{\text{Head}}(\text{SUPERVISION} \bowtie_{\text{Employee = Number}} (\text{R1}))) = \text{R2}$$

Head
301
375

**EMPLOYEES**  $\bowtie_{\text{Number=Head}}$  **(R2)**

Head	Number	Name	Age	Salary
301	301	Samir Salem	34	70
375	375	Mona Salem	50	65

$\pi_{\text{Number, Name}}$  (**EMPLOYEES**  $\bowtie_{\text{Number=Head}}$  **(R2)**)

Number	Name
301	Samir Salem
375	Mona Salem



# Algebra With Null Values

People

Name	Age	Salary
Aldo	35	15
Andrea	27	21
Maria	NULL	42

- $\sigma_{\text{Age} > 30}(\text{People})$
- which tuples belong to the result?
- The first yes, the second no, but the third????
  - **True (T) → Result**
  - **False (F)**
  - **Unknown (U)**

Name	Age	Salary
Aldo	35	15

# Quiz

- Show the output of theses two queries:

$\sigma_{\text{Mark} \geq 7 \wedge \text{Age} \leq 18}(\text{STUDENTS})$

$\sigma_{\text{Mark} \geq 7}(\sigma_{\text{Age} \leq 18}(\text{STUDENTS}))$

## STUDENTS

RegNum	Name	Mark	Age
105	Noha Kamal	7	18
107	Maged Mostafa	8	17
110	Samir Shawky	6	17
132	Dina Hady	9	19
154	Ahmed Amin	8	20

$\sigma_{F1 \wedge F2}(R) = \sigma_{F1}(\sigma_{F2}(R))$

RegNum	Name	Mark	Age
105	Noha Kamal	7	18
107	Maged Mostafa	8	17

# Relational Calculus

- Expressions have this form:

$$\{A_1:x_1,\dots,A_k:x_k|f\}$$

Where:

- $A_1,\dots,A_k$  are distinct attributes
- $x_1,\dots,x_k$  are variables
- $f$  is a formula:
  - Comparison operators  
 $=, <>, <, >$ , etc.
  - Logical connectives
    - $\neg$  - not
    - $\wedge$  - and
    - $\vee$  - or
  - Quantifiers
    - $\forall X(p(X))$ : For every  $X$ ,  $p(X)$  must be true
    - $\exists X(p(X))$ : There exists at least one  $X$  such that  $p(X)$  is true

# Relational Algebra and Calculus

## Relational Algebra

- Theoretical foundation for SQL
- Higher level than programming language  
**but still must specify steps to get desired result**

## Relational Calculus

- Formal foundation for Query-by-Example
- A first-order logic description of desired result
- **Only specify desired result, not how to get it**

# Example1:

- **Write a relational calculus expression for the following query:**
  - Find the numbers, names and ages of employees earning more than 40 thousands.

## Relational algebra:

$\pi_{\text{Number, Name, Age}} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES}))$

## Relational Calculus:

$\{ \text{Number:m, Name:n, Age:a} \mid$   
 $\text{EMPLOYEES}(\text{Number:m, Name:n, Age:a,}$   
 $\text{Salary:s}) \wedge s > 40 \}$

Number	Name	Age
104	Amr Khalil	38
210	Medhat Aly	49
231	Samia Elkholy	50
252	Ihab Zohdy	44
301	Samir Salem	34
375	Mona Salem	50

# Example1:

- **Write a relational calculus expression for the following query:**
  - Find the numbers, names and ages of employees earning more than 40 thousands.

**Relational algebra:**

$$\pi_{\text{Number, Name, Age}} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES}))$$

**Relational Calculus:**

$$\begin{aligned} & \{ \text{Number} : m, \text{Name} : n, \text{Age} : a \mid \\ & \text{EMPLOYEES} (\text{Number} : m, \text{Name} : n, \text{Age} : a, \text{Salary} : s) \wedge s > 40 \} \\ & \text{OR} \{ \text{Number} : m, \text{Name} : n, \text{Age} : a \mid \\ & \exists s (\text{EMPLOYEES} (\text{Number} : m, \text{Name} : n, \text{Age} : a, \text{Salary} : s) \wedge s > 40) \} \end{aligned}$$



# Example 2

- **Write a relational calculus expression for the following query:**
  - Find the registration numbers of the supervisors of the employees earning more than 40 thousands.

**Relational algebra:**

$$\pi_{\text{Head}} (\text{SUPERVISION} \bowtie_{\text{Employee} = \text{Number}} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES})))$$

**Relational Calculus:**

$$\{ \text{Head} : h \mid \text{EMPLOYEES}(\text{Number} : m, \text{Name} : n, \text{Age} : a, \text{Salary} : s) \wedge \\ \text{SUPERVISION}(\text{Employee} : m, \text{Head} : h) \wedge s > 40 \}$$

# Example 3

- Write a relational calculus expression for the following query:
  - Find the names and salaries of the supervisors of the employees earning more than 40 thousands.

Relational algebra:

$$\pi_{\text{NameH, SalaryH}} (\rho_{\text{NumberH, NameH, AgeH, SalaryH} \leftarrow \text{Number, Name, Age, Salary}} (\text{EMPLOYEES})$$

$$\bowtie_{\text{NumberH=Head}} (\text{SUPERVISION} \bowtie_{\text{Employee = Number}} (\sigma_{\text{Salary} > 40} (\text{EMPLOYEES})))$$

Relational Calculus:

$$\{ \text{NameH: nh, SalaryH: sh} \mid \text{EMPLOYEES}(\text{Number: m, Name: n, Age: a, Salary: s})$$

$$\wedge \text{EMPLOYEES}(\text{NumberH: h, NameH: nh, AgeH: ah, SalaryH: sh}) \wedge$$

$$\text{SUPERVISION}(\text{Employee: m, Head: h}) \wedge s > 40 \}$$

# Example 4

- **Write a relational calculus expression for the following query:**
  - Find the employees earning more than their respective supervisors, showing registration numbers, names and salaries of the employees and supervisors.

**Relational algebra:**

$$\pi_{\text{Number, Name, Salary, NumberH, NameH, SalaryH}} \left( \sigma_{\text{Salary} > \text{SalaryH}} \left( \rho_{\text{NumberH, NameH, SalaryH, AgeH} \leftarrow \text{Number, Name, Salary, Age}} (\text{EMPLOYEES}) \right. \right. \\ \left. \bowtie_{\text{NumberH=Head}} (\text{SUPERVISION}) \bowtie_{\text{Employee = Number}} (\text{EMPLOYEES}) \right)$$

**Relational Calculus:**

$$\{ \text{Number: m, Name: n, Salary: s, NumberH: h, NameH: nh, SalaryH: sh} \mid \\ \text{EMPLOYEES}(\text{Number: m, Name: n, Age: a, Salary: s}) \wedge \\ \text{EMPLOYEES}(\text{Number:h, Name: nh, Age: ah, Salary: sh}) \wedge \\ \text{SUPERVISION}(\text{Employee: m, Head: h}) \wedge s > sh \}$$

*Thanks*